Sri Lanka Institute of Information Technology

IT0060 –Essential Mathematics

Ms. Pubudika Wijesundara
Ms. Shashini Kumarasinghe

Integration

Outline

- Indefinite integrals and notations
- Basic integration rules
- Integration of polynomials
- Definite integrals
- Area under a curve

Indefinite integrals and notations

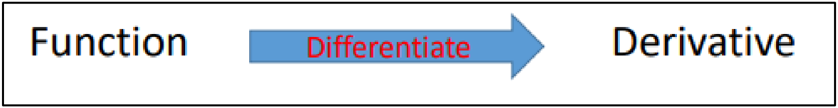
Integration as reverse operation of differentiation

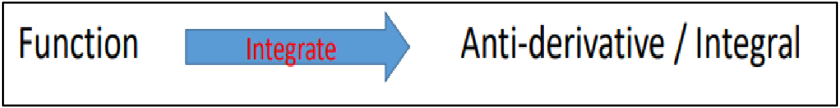
In Differential Calculus we were given a function, $f(x)$, and we learned how to find the derivative of this function.

Now we are going to turn things around. We now want to ask what function we differentiated to get the function $f(x)$.

Indefinite integrals and notations

Differentiation Vs Integration

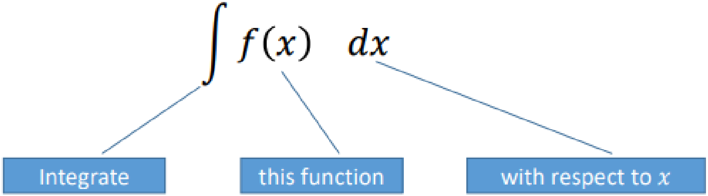




Indefinite integrals and notations

Suppose you want to integration a function $f(x)$.

This can be written as,



Indefinite integrals and notations

When we integrate a function, we do not get a definite anti-derivative. This is because there are infinitely many functions with the same shape with different y-intercepts that have the same derivative function.

Therefore, when integrating we must add an unknown constant C. This is known as the constant of integration.

This integration is called indefinite integration.

Basic integration Rules

	Function $f(x)$	Anti-Derivative function $\int f(x)dx$
Constant rule	$f(x) = k$	kx k *represents any real number*
Power rule	$f(x) = x^n$ $n \neq -1$	$\frac{x^{n+1}}{n+1} + C$
e rule	$f(x) = e^x$	$e^x + C$
***ln* rule**	$f(x) = \frac{1}{x}$	$\ln x + C$
Sin rule	$f(x) = cos\ x$	$sin\ x + C$
Cos rule	$f(x) = \sin x$	$-\cos x + C$
Tan rule	$f(x) = \sec^2 x$	$tan\ x + C$

Basic integration Rules Examples

Find the following indefinite integrals.

$\int x^5\,dx$ >> $\frac{x^{5+1}}{5+1} + c = \frac{x^6}{6} + c$

$\int x^{-2}\,dx$ >> $\frac{x^{-2+1}}{-2+1} + c = \frac{x^{-1}}{-1} + c$

$\frac{1}{x} + c$

Basic integration Rules Examples

Find the following indefinite integrals.

$$\int \sqrt{x}\, dx \qquad >> \qquad \int x^{\frac{1}{2}}\, dx$$

$$\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\frac{2}{3} x^{\frac{3}{2}} + c$$

Basic integration Rules

	Function y	Derivative function $\int y\,dx$
Constant multiple rule	$y = kf(x)$	$\int y\,dx = k\int f(x)\,dx$
Sum rule	$y = f(x) + g(x)$	$\int y\,dx = \int f(x)\,dx + \int g(x)\,dx$
Difference rule	$y = f(x) - g(x)$	$\int y\,dx = \int f(x)\,dx - \int g(x)\,dx$

Basic integration Rules Examples

Find the following indefinite integrals.

$$\int (2x^3 - 3x + 4)\, dx \quad >> \quad \frac{2x^{3+1}}{3+1} - \frac{3x^2}{2} + 4x + c$$

$$\frac{2x^4}{4} - \frac{3x^2}{2} + 4x + c$$

$$\frac{x^4}{4} - \frac{3x^2}{2} + 4x + c$$

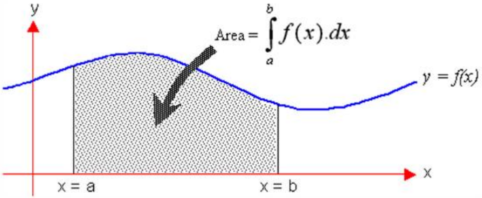
Definite integrals

The definite integral of any function can be expressed either as the limit of a sum or if there exists an antiderivative F for the interval [a, b], then the definite integral of the function is the difference of the values at points a and b (a is the lower limit and b is the upper limit).

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Area under a curve

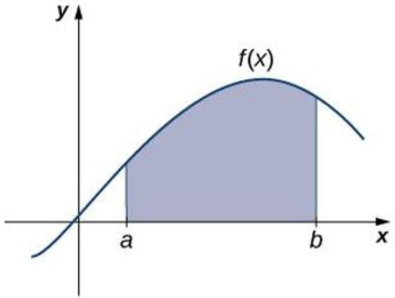
The process of integration (finding the anti-derivative) is quite ‘magically’ connected to another entirely different process. i.e. finding the area under a curve. The area under a curve (a function) between two points a and b can be found using the anti-derivative of the function evaluated at those two points.



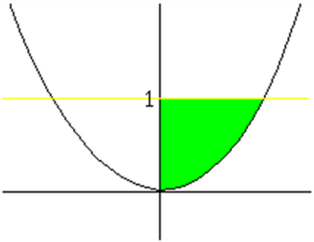
Area under a curve

Integration can be used to find the area:

Between a curve and the x-axis

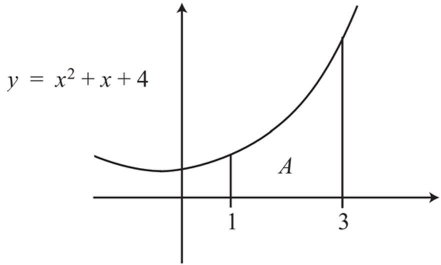


Between a curve and the y-axis



Area Under a Curve Example

Suppose you want to find the area below a function y $= x^2 + x + 4$ from $x = 1$ to $x = 3$.



$$A = \int_1^3 y \, dx$$

$$= \int_1^3 (x^2 + x + 4) \, dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} + 4x \right]_1^3$$

$$= \left[\frac{27}{3} + \frac{9}{2} + 12 \right] - \left[\frac{1}{3} + \frac{1}{2} + 4 \right]$$

$$= 20.667$$

Thank You!